Comparing Different Overlay Topologies and Metrics in Pulse-Coupled Multi-Agent Systems

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Abstract. In Multi-Agent Systems (MASs) it is of vital importance that basic network operations are performed without the interference of a central entity (i.e. agent). In this paper we will present how to use a self-organization approach to achieve time synchronization of agents in MASs using a Pulse-Coupled Oscillators (PCO) model that is based on flashing fireflies. Fireflies are known to emit flashes at regular intervals when isolated, but when they are within a group, they converge upon the same rhythm until time synchronization is achieved. This paper investigates how the choice of overlay network topology and metric affects the time synchronization process of agents in MASs.
Keywords: self-organization, firefly synchronization, pulse-coupled oscillators, MASON

1 Introduction

In distributed systems, such as Multi-Agent Systems (MASs), time synchronization is a prerequisite for many processes (e.g. maintaining the consistency of distributed data or checking the authenticity of a request sent to the server). In this paper we present a self-organization approach for time synchronization of agents in MASs using a Pulse-Coupled Oscillators (PCO) model that is based on flashing fireflies. Self-organization is a process in which patterns at the global level of a system emerge solely from numerous interactions among lower-level components in the system [6]. This type of synchronization is applicable in MASs since agents can self-organize themselves and no external control is needed.

The focus in this paper is on investigation how different overlay network topologies and metrics affect the time synchronization process of agents in MASs (i.e. which parameter has a greater impact on the time synchronization process). The choice of an overlay network topology determines the way agents are coupled (i.e. are connected), while the usage of a metric denotes the intensity of influence that coupled agents have on each other.

The rest of this paper is organized as follows. Section 2 introduces the PCO model for time synchronization explaining the meaning of coupling constants and functions. Section 3 gives a short overview of related work and presents our previous work. Furthermore, Section 4 explains the simulation parameters, the evaluation criteria and presents the results. Finally, Section 5 concludes the paper and outlines directions for future work.

2 The Pulse-Coupled Oscillators model

Time synchronization of agents in MASs can be achieved using the PCO model and can be described with the set $PCO_{MAS} = \{\alpha,\ \phi,\ \Delta,\ \tau,\ \mu\}$. $\alpha = \{a_\imath \mid \imath \in \mathbb{N}\}$ denotes the set of agents in the MAS. The set $\phi = \{\varphi_\imath \mid \imath \in \mathbb{N};\ \varphi_\imath \in [0,\ 1]\}$ contains agents' internal phases. Furthermore, the set $\Delta = \{(x_\imath,\ y_\imath) \mid \imath \in \mathbb{N};\ x_\imath,\ y_\imath \in \mathbb{R}^+\}$ contains all agents' coordinates. Finally, the set $\tau = \{$*fully-meshed, line, meshed, ring, star*$\}$ contains different topologies, while the set $\mu = \{Chebyshev,\ Euclidean,\ Mahalanobis,\ Manhattan\}$ contains different metrics that are used in order to calculate distances among agents. In MASs the dynamic behavior of agent's $a_\imath$ oscillator is given by the following equation

$$z_\imath(t) = f(\varphi_\imath) + \sum_{\jmath=1}^{N} \varepsilon_{\imath\jmath}\ g_{\imath\jmath}(t), \tag{1}$$

where $z_\imath(t)$ is a value of a state variable $z_\imath$ at the moment t, $\varphi_\imath$ denotes agent's $a_\imath$ internal phase, $f(\varphi_\imath)$ describes the *excitation* evolution of agent's $a_\imath$ oscillator, N denotes the number of agents in the MAS, $\varepsilon_{\imath\jmath}$ is a coupling constant, while $g_{\imath\jmath}(t)$ is a coupling function between agents' $a_\imath$ and $a_\jmath$ oscillators.

Theorem 1. ***The Mirollo and Strogatz theorem (1990)*** — *If the function* $f(\varphi_\imath) : [0,\ 1] \to [0,\ 1]$ *is smooth, monotonically increasing and concave down, then the set of fully-meshed connected agents' oscillators will always converge to synchronicity for any arbitrary set of agents and initial conditions [11].*

The coupling constant $\varepsilon_{\imath\jmath}$ denotes the intensity of the influence that coupled agents $a_\imath$ and $a_\jmath$ have on each other. Mirollo and Strogatz assumed that the coupling constant $\varepsilon_{\imath\jmath}$ is equal for all agents (i.e. $\varepsilon_{\imath\jmath} = \varepsilon,\ \forall\imath\ \forall\jmath$) and proved that synchronization can be achieved if $\varepsilon \in \langle 0,\ 1\rangle$ [11]. In 2010 Zhulin et al. proved that synchronization can be achieved even in the conditions of different coupling constants that are distributed in a close interval [1].

Theorem 2. ***The Zhulin et al. theorem (2010)*** — *Given two oscillators* $a_\imath$ *and* $a_\jmath$ *with their coupling strengths satisfying* $\varepsilon_{\imath\jmath} \neq \varepsilon_{\jmath\imath}$, *they will achieve synchronization [1].*

The coupling function $g_{\imath\jmath}(t)$ denotes whether agents $a_\imath$ and $a_\jmath$ are coupled and is calculated as

$$g_{\imath\jmath}(t) = \begin{cases} 1, & \text{if } a_\imath \text{ is coupled with } a_\jmath \text{ and } t = t^*_\jmath \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where $t^*_\jmath$ is the firing time of agent's $a_\jmath$ oscillator. Mirollo and Strogatz assumed that every agent is coupled with all other agents in the system (i.e. fully-meshed connectivity) [11]. Almost 15 years after their seminal work, Lucarelli and Wang showed that the fully-meshed assumption can be replaced with the partly-meshed assumption and that agents still can achieve time synchronization [9].

2.1 Uncoupled oscillator

When agent's a_i oscillator is not coupled, its state variable z_i changes following its own *excitation* described with the function $f(\varphi_i)$. Fig. 1 shows an example of the temporal evolution of agent's a_i oscillator *excitation*. When the state variable z_i reaches the *threshold*, agent's a_i oscillator "fires" and then its state variable z_i jumps back to 0 (i.e. $z_i \in [0, threshold]$). In the example shown in Fig. 1, the function $f(\varphi_i)$ is a line and the value of *threshold* is set to 1.

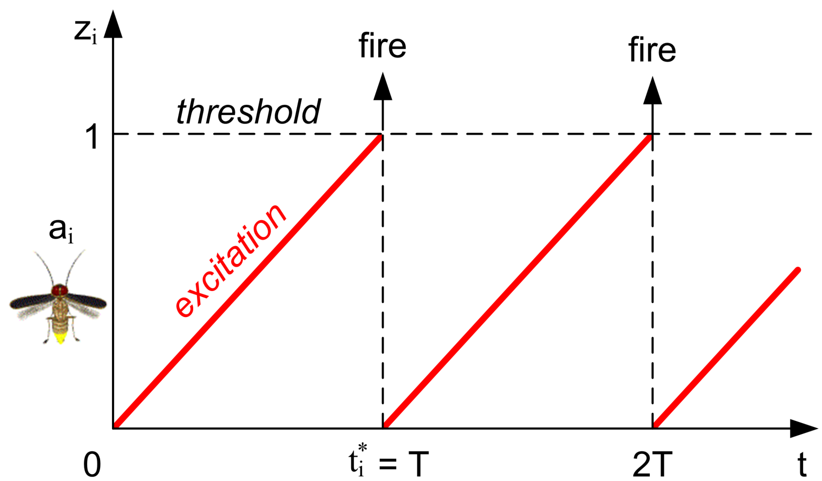

Fig. 1. An example of the temporal evolution of agent's a_i oscillator *excitation*

2.2 Two coupled oscillators

The PCO model is based on the experiments conducted on real fireflies by Buck et al. in 1981 [5]. Agent's a_i state variable z_i changes as follows

$$z_i(t_j^*) = \begin{cases} threshold, & \textit{if the condition } c \textit{ is met} \\ f(\varphi_i), & \textit{otherwise} \end{cases} . \tag{3}$$

The condition c is met if

$$z_i(t_j^{*-}) \;<\; threshold - buffer \tag{4}$$

and

$$z_i(t_j^{*-}) + f(\varphi_i) + \sum_{j=1}^{N} \varepsilon_{ij}\; g_{ij}(t_j^*) \;>\; trigger \tag{5}$$

where t_j^{*-} denotes the moment before the firing time t_j^* of agent's a_j oscillator.

Fig. 2 shows that at the moment t_j^* condition c is met for agent $a_\imath$. Thus, agent $a_\imath$ sets its state variable $z_\imath$ to the *threshold* and then flashes. The condition c is met if its state variable $z_\imath$ at the moment t_j^{*-} is smaller than the value of the expression $(\mathit{threshold} - \mathit{buffer})$ (Equation (4)) and if the amount of luminescence from the agents that are coupled with agent $a_\imath$ plus agent's $a_\imath$ *excitation* is larger than a *trigger* (Equation (5)). Otherwise, agent $a_\imath$ will not be influenced by the agents that it is coupled with (see Fig. 2 at the moment $t_\imath^*$).

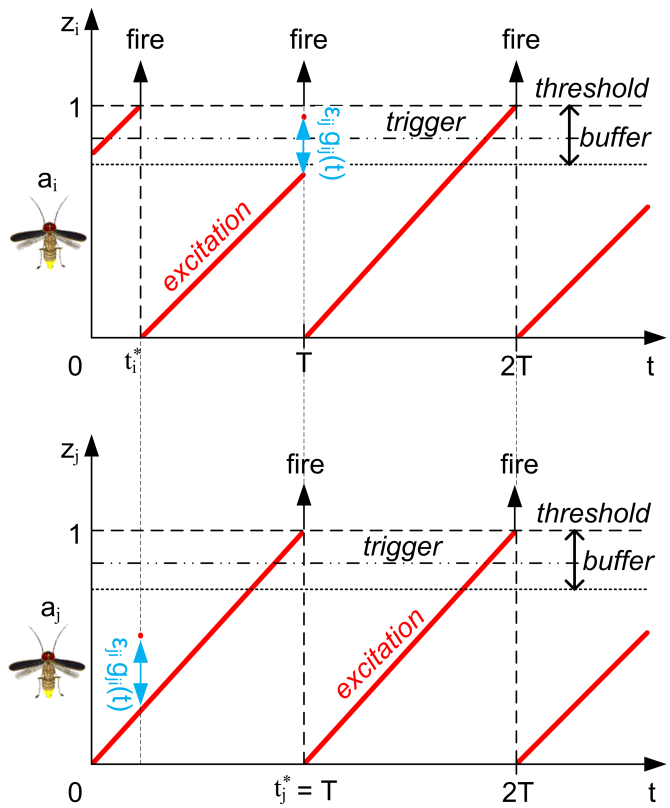

Fig. 2. Example of the temporal evolution of agents $a_\imath$ and $a_\jmath$ oscillators

The coupling function $g_{\imath\jmath}(t)$. In order to calculate the coupling function $g_{\imath\jmath}(t)$ it is necessary to find whether agent $a_\jmath$ is coupled with agent $a_\imath$ at the moment t using information from an overlay network topology. The overlay network topology is defined by agents' coordinates from the set Δ, a type of the overlay network topology from the set τ and a metric from the set μ. In this paper we use fully-meshed, meshed, ring, line and star overlay network topologies (for more details about overlay network topologies see [3]).

The coupling constant ε_{ij}. The coupling constant ε_{ij} is inversely proportional to the distance between agents a_i and a_j in the chosen metric. In our previous work [3] [4] we calculated the coupling constant ε_{ij} using only *Euclidean* distance. In this paper we extended the set μ with three more elements: *Chebyshev*, *Mahalanobis* and *Manhattan* distances.

Chebyshev distance

Chebyshev distance is defined on a vector space where the distance between agents a_i and a_j is the greatest of all of their differences along any coordinate dimension. The mathematical expression for *Chebyshev* distance is

$$d_c(a_i, a_j) = \max(\mid x_i - x_j \mid, \ \mid y_i - y_j \mid). \tag{6}$$

Euclidean distance

Euclidean distance between agents a_i and a_j is the length of the line segment connecting them. *Euclidean* distance can be calculated as

$$d_e(a_i, a_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}. \tag{7}$$

Mahalanobis distance

When agents a_i and a_j are independent, then *Mahalanobis* distance becomes normalized *Euclidean* distance and can be calculated as

$$d_h(a_i, a_j) = \sqrt{\frac{(x_i - x_j)^2}{var(X)} + \frac{(y_i - y_j)^2}{var(Y)}}. \tag{8}$$

where $var(X)$ is the variance of all agents' x_i, $\forall i$ coordinates calculated as

$$\begin{aligned} var(X) &= \frac{1}{N-1} \textstyle\sum_{i=1}^{N} (x_i - \xi_x)^2 \\ \xi_x &= \frac{1}{N} \textstyle\sum_{j=1}^{N} x_j \end{aligned} \tag{9}$$

and $var(Y)$ is the variance of all agents' y_i, $\forall i$ coordinates calculated as

$$\begin{aligned} var(Y) &= \frac{1}{N-1} \textstyle\sum_{i=1}^{N} (y_i - \xi_y)^2 \\ \xi_y &= \frac{1}{N} \textstyle\sum_{j=1}^{N} y_j. \end{aligned} \tag{10}$$

Manhattan distance

Manhattan distance is based on calculating the distance between agents a_i and a_j as a sum of absolute differences of their coordinates. The mathematical expression for *Manhattan* distance is

$$d_n(a_i, a_j) = \mid x_i - x_j \mid + \mid y_i - y_j \mid. \tag{11}$$

3 Related work

The Mirollo and Strogatz model [11] gave a theoretical framework for the convergence to synchrony in fully-meshed networks, while Lucarelli and Wang showed that the algorithms developed by Mirollo and Strogatz would converge for any connected topology [9]. Hence, the Mirollo and Strogatz model considers that there exists physical connectivity among all agents in the system, while the Lucarelli and Wang model specifies the interconnection in the network with a graph in which edges join connected agents.

In our previous work [3] [4] we investigated what is the effect that different overlay network topologies (i.e. fully-meshed, meshed, ring, line and star) have on the time synchronization process. This paper extends our previous work by adding different metrics denoting a *credibility* measure. These two parameters have already been studied in related work, but to our knowledge, no previous study compared their mutually effect on the time synchronization process.

3.1 Overlay network topologies

The usage of the PCO model for time synchronization can be found in various algorithms (e.g. geographic routing algorithms [12] and the Reachback Firefly Algorithm (RFA) [15]), protocols (e.g. gossip protocols [16]) or data gathering mechanisms (e.g. data gathering scheme in sensor networks [14]). All of the aforementioned examples assumed meshed networks in which all agents were the same and had the same influence on each other (i.e. $\varepsilon_{ij} = \varepsilon$).

In every step of a geographic routing [12], an agent can communicate only with its k-neighbors, referred to as the Connected k-Neighborhood (CKN) problem. This k-connectivity is also used in [2], where for every agent, k-neighbors are selected randomly. In the RFA four different topologies have been investigated: fully-meshed and meshed [15], chain [8] and ring [7]. In the chain topology, agents are ordered in a chain and can communicate only with their immediate neighbors, while the ring topology was simulated using asynchronous communication patterns (i.e. unidirectional communication links).

3.2 *Credibility* measures

Although, in most related work the equal *credibility* is assumed for all agents, there are projects where the coupling constant ε_{ij} is different for different agents. The usage of the *credibility* measure enables different agents to have different influence on others. When agents (e.g. attackers) are less credible, then they have less influence on other agents. Therefore, the *credibility* measure can be used as a means to increase the robustness of the time synchronization process.

In [13] authors proposed that agent's *credibility* depended on the degree-based weighting. Being influenced by more agents means that the agent is likely to be more reliable. Furthermore, Zhulin et al. assumed that all agents' coupling constants stayed constant during the time synchronization process and were distributed within a close interval. They proved that under those conditions time synchronization can be achieved [1].

4 The simulation of time synchronization in the MAS

Our simulation is run using a single-process discrete-event simulation core and visualization toolkit written in Java called Multi-Agent Simulator of Neighborhoods (MASON). In this paper, we will not explain how agents' activity is scheduled in the simulator, nor other details about the simulation process, since it can be find in the MASON's documentation [10].

4.1 Simulation rules

The behavior of each agent in simulation is composed of two parts: *active* and *passive* one. In the active part, the value of state variable $z_\imath$ increases until it reaches the *threshold* (i.e. $z_\imath = f(\varphi_\imath)$), at which point a flash is emitted and then $z_\imath = 0$. Since the flashes are timed by the progressive excitation (described with the function $f(\varphi_\imath)$) within each agent, when left alone, each agent flashes at periodic intervals. In the passive part, each agent senses a certain amount of luminescence from the agents that it is coupled with and reacts upon it according Equations (3), (5) and (4).

4.2 Simulation parameters

The simulation parameters include *numberFireflies*, *threshold* and *buffer*. The *numberFireflies* parameter sets the number of fireflies in the simulation. Furthermore, the *threshold* parameter denotes at which point an agent will flash and reset its state variable to zero. Finally, the *buffer* parameter sets how many time steps are necessary for the flashing signal to evolve and terminate in a flash. If an agent is triggered to reset its state variable while condition from Equation (4) is not met, the flash will proceed as planned despite the resetting.

In this paper we simulate small overlay networks with ten agents, and therefore the *numberFireflies* parameter is set to 10. In order to determine values of *threshold* and *buffer* parameters, in our previous work [3] we made simulation experiments. We changed the ratio between *threshold* and *buffer* parameters from 0.1 to 0.9. The conclusion was that the best ratio is 0.1. This means that the relationship between *threshold* and *buffer* parameters can be described with the following equation: $buffer = 0.1\ threshold$. Therefore, in this paper we choose that the *threshold* value is equal to 20 and the *buffer* value is equal to 2.

4.3 Evaluation criteria

The evaluation criteria are the same as in our previous papers [3] [4]: the percentage of time synchronization success, the network traffic and the time needed to achieve time synchronization. Agents are said to be synchronized when they flash simultaneously. The network traffic denotes the amount of messages exchanged among agents during the time synchronization process, while the time needed to achieve time synchronization denotes discrete time units (i.e. steps) until all agents are synchronized.

4.4 Simulation results

In our previous work [3] we used only *Euclidean* distance with combination of five different overlay network topologies and concluded that best topologies on condition of the percentage of time synchronization success are fully-meshed, meshed and star topologies. In this work we extend our previous results using new metrics. For the purpose of testing the influence of topologies and metrics on the time synchronization process we used combinations of all metrics and topologies (i.e. forty combinations). Each combination is executed 100 times and then average results are shown in Tables 1 and 2.

First we investigate how different metrics affect the percentage of time synchronization success. Results in Table 1 show that *Manhattan* distance is resulting with the best result. For *mesh(3)* topology, when using *Manhattan* distance, results are 10% better than when using *Chebyshev* distance.

Furthermore, we investigate which parameter is more important: the overlay network topology or the metric. From Table 1 it can be concluded that the choice of overlay network topology has a greater influence on the percentage of time synchronization success than the chosen metric. The most significant difference between different overlay network topologies is between fully-meshed and ring topologies when using *Euclidean* distance (27%). The most significant difference between different metrics is between *Chebyshev* and *Manhattan* distances for *mesh(4)* topology (12%).

Finally, we measure the time needed to achieve time synchronization and the number of exchanged messages considering the usage of different combinations of overlay network topologies and metrics. Form the results shown in Table 2, we can conclude that the choice of metric does not have a significant impact on those two parameters.

Table 1. The percentage of time synchronization success

		Metrics			
		Chebyshev	Euclidean	Mahalanobis	Manhattan
Topologies	Fully-meshed	94%	97%	94%	96%
	Line	70%	69%	72%	72%
	Mesh(3)	74%	78%	78%	84%
	Mesh(4)	77%	84%	84%	89%
	Mesh(5)	91%	93%	93%	93%
	Mesh(6)	89%	92%	92%	93%
	Mesh(7)	88%	94%	94%	96%
	Mesh(8)	96%	93%	93%	95%
	Ring	71%	70%	71%	72%
	Star	94%	94%	94%	94%

Table 2. Average time needed to achieve time synchronization / average network traffic during the time synchronization process

		Metrics			
		Chebyshev	Euclidean	Mahalanobis	Manhattan
Topologies	Fully-meshed	47/217	42/198	47/217	43/202
	Line	128/83	128/83	128/83	128/83
	Mesh(3)	64/76	64/77	64/77	68/83
	Mesh(4)	54/92	56/97	56/97	56/98
	Mesh(5)	54/124	54/123	54/123	53/120
	Mesh(6)	49/139	48/136	48/136	46/130
	Mesh(7)	47/159	46/157	46/157	46/158
	Mesh(8)	49/197	45/183	45/183	44/175
	Ring	95/70	95/70	95/70	95/70
	Star	40/29	40/29	40/29	40/29

5 Conclusions and future work

This paper proposes a time synchronization scheme for Multi-Agent Systems (MASs) using fireflies as role models. In nature fireflies are known to emit flashes at regular intervals when isolated, while in a group they achieve synchronization. In computer science, fireflies behavior is modeled with a Pulse-Coupled Oscillators (PCO) model. This model can be used in MASs since every agent can run an algorithm similar to the one run by fireflies in nature. Moreover, the PCO model is said to be robust, scalable and adaptive, all of which are important in MASs.

In this paper we tested our algorithm with different overlay network topologies and metrics. Information from overlay network topologies is used in order to find coupled agents, while metrics are used in order to calculate the intensity of influence that coupled agents have on each other. From results presented in this paper, it can be concluded that time synchronization can be achieved when using different coupling constants and moreover, that the choice of overlay network topology is more important than the choice of used metric.

The goal of my Ph.D. research is to design a firefly algorithm that shows a high percentage of robustness without a significant decrease in the percentage of time synchronization success. In order to achieve my goal I will use the coupling constant ε_{ij} as a parameter that denotes agents' *credibility* such that some agents (e.g. attackers) are less credible and thus have less influence on other agents. Therefore, the focus in this paper was on investigation of how coupling constants affected the time synchronization process.

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References

1. An, Z., et al.: Linear Pulse-Coupled Oscillators Model: A New Approach for Time Synchronization in Wireless Sensor Networks. Advances in Complex Systems 2(2), 108–114 (2010)
2. Babaoglu, O., et al.: Firefly-Inspired Heartbeat Synchronization in Overlay Networks. In: Proceedings of Self-Adaptive and Self-Organizing Systems Conference. pp. 77–86 (2007)
3. Bojic, I., et al.: A Self-Optimizing Mobile Network: Auto-Tuning the Network with Firefly-Synchronized Agents. Information Sciences 182(1), 77–92 (2012)
4. Bojic, I., Kusek, M.: Fireflies Synchronization in Small Overlay Networks. In: Proceedings of Information and Communication Technology, Electronics and Microelectronics Conference. pp. 27–32 (2009)
5. Buck, J., et al.: Control of Flashing in Fireflies. Journal of Comparative Physiology A 144, 287–298 (1981)
6. Camazine, S., et al.: Self-Organization in Biological Systems. Princeton University Press (2003)
7. Leidenfrost, R., Elmenreich, W.: Establishing Wireless Time-Triggered Communication Using a Firefly Clock Synchronization Approach. In: Proceedings of Intelligent Solutions in Embedded Systems Workshop. pp. 1–18 (2008)
8. Leidenfrost, R., Elmenreich, W.: Firefly Clock Synchronization in an 802.15.4 Wireless Network. EURASIP Journal on Embedded Systems 2009(7), 1–17 (2009)
9. Lucarelli, D., Wang, I.J.: Decentralized Synchronization Protocols with Nearest Neighbor Communication. In: Proceedings of Embedded Networked Sensor Systems Conference. pp. 62–68 (2004)
10. MASON web site: http://www.cs.gmu.edu/ eclab/projects/mason/
11. Mirollo, R.E., Strogatz, S.H.: Synchronization of Pulse-Coupled Biological Oscillators. SIAM Journal on Applied Mathematics 50(6), 1645–1662 (1990)
12. Nath, S., Gibbons, P.: Communicating via Fireflies: Geographic Routing on Duty-Cycled Sensors. In: Proceedings of Information Processing in Sensor Networks Conference. pp. 440–449 (2007)
13. Scholtes, I., et al.: Epidemic Self-Synchronization in Complex Networks of Kuramoto Oscillators. Advances in Complex Systems 13(1) (2010)
14. Taniguchi, Y., Wakamiya, N., Murata, M.: A Distributed and Self-Organizing Data Gathering Scheme in Wireless Sensor Networks. In: Proceedings of Information and Telecommunication Technologies Symposium. pp. 299–304 (2005)
15. Werner-Allen, G., et al.: Firefly-Inspired Sensor Network Synchronicity with Realistic Radio Effects. In: Proceedings of Embedded Networked Sensor Systems Conference. pp. 142–153 (2005)
16. Wokoma, I., et al.: A Weakly Coupled Adaptive Gossip Protocol for Application Level Active Networks. In: Proceedings of Policies for Distributed Systems and Networks Workshop. pp. 244–247 (2002)